

행렬과 기하학적 변환

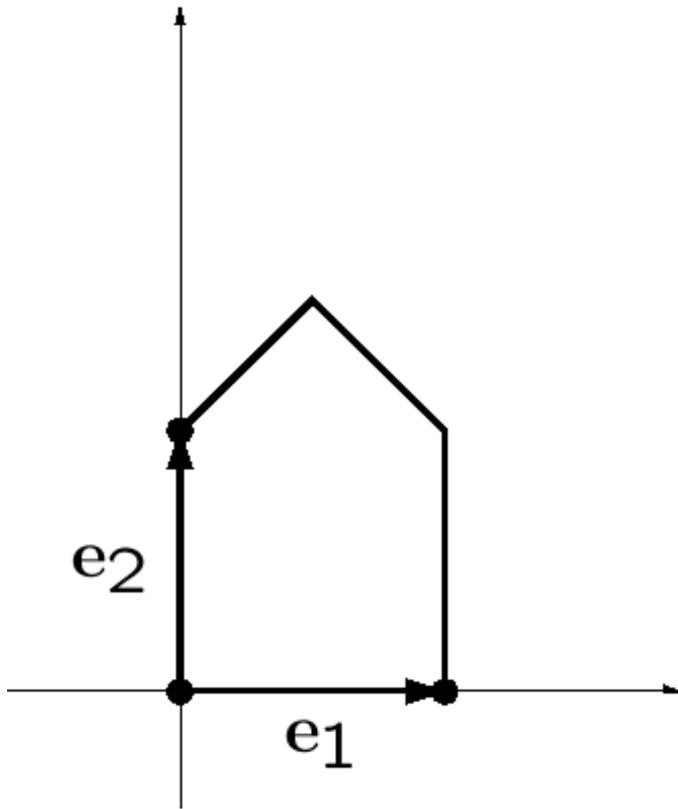
서울대학교 컴퓨터공학부

김명수

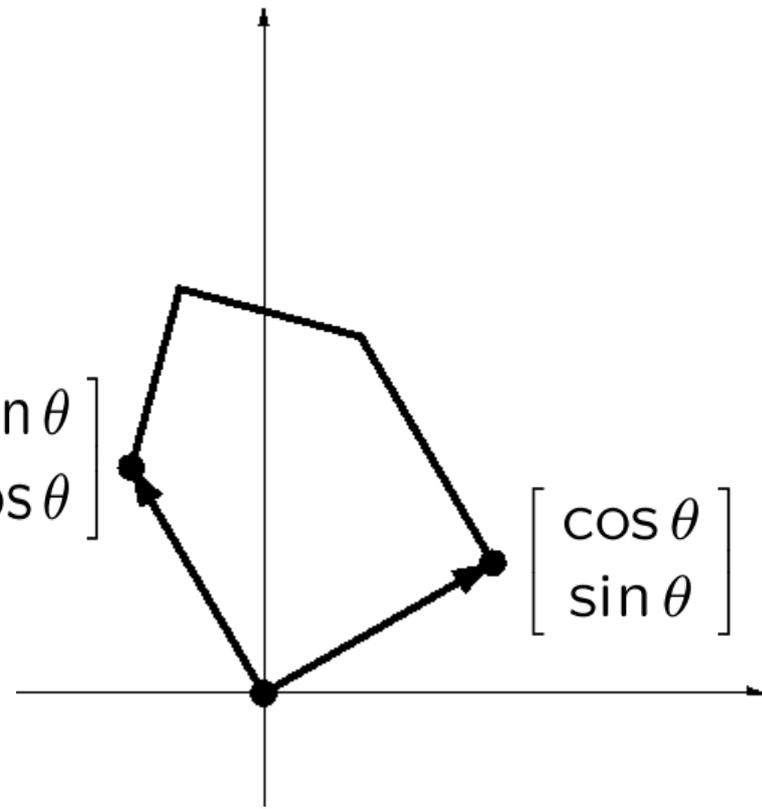
<http://cse.snu.ac.kr/mskim>

<http://3map.snu.ac.kr>

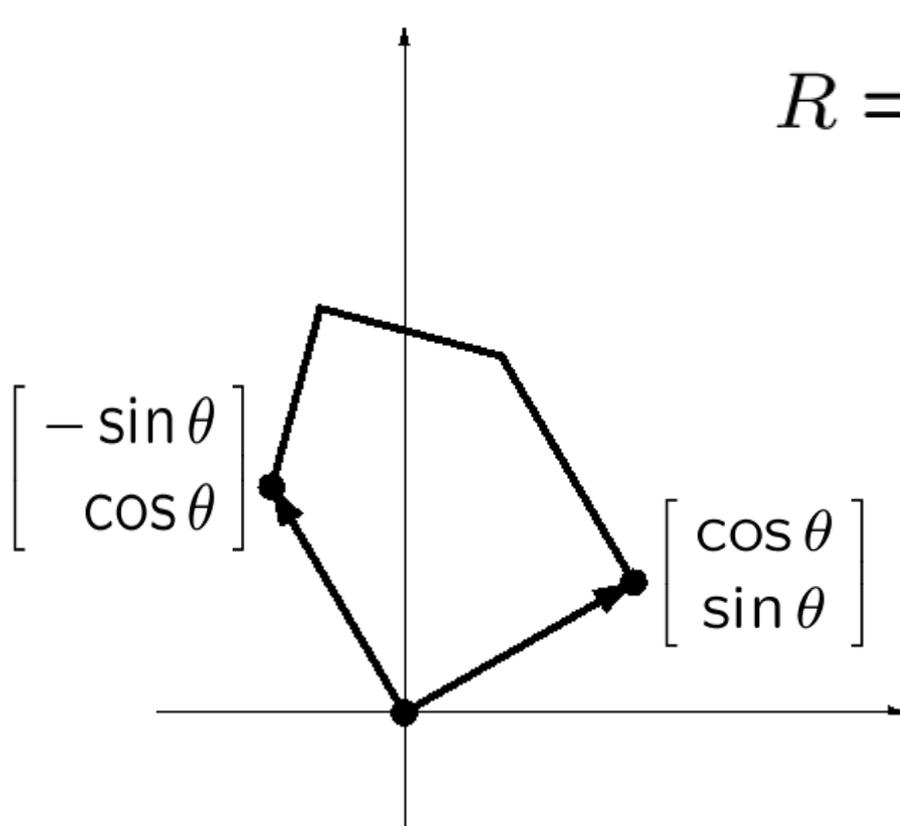
2차원 회전



$$\begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$



2차원 회전

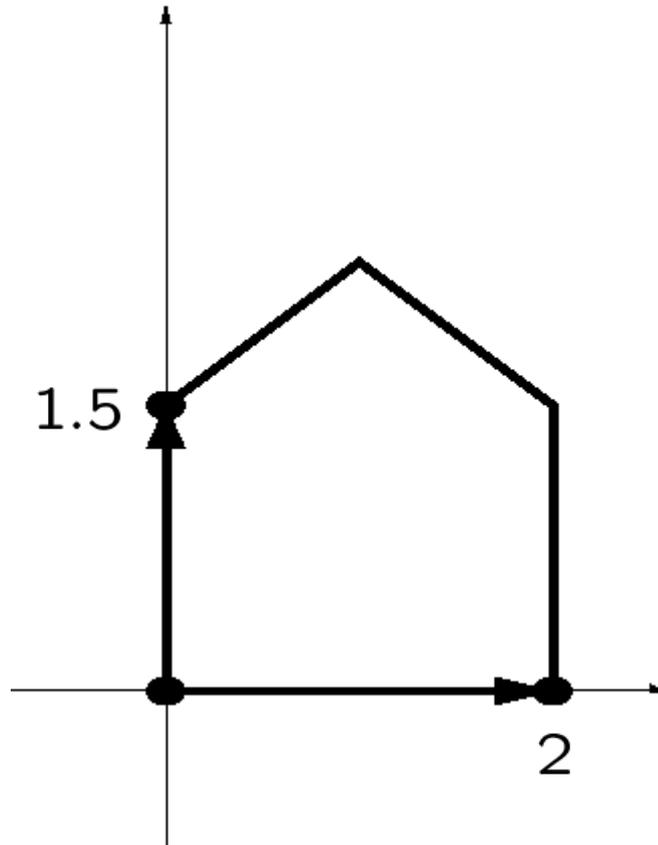
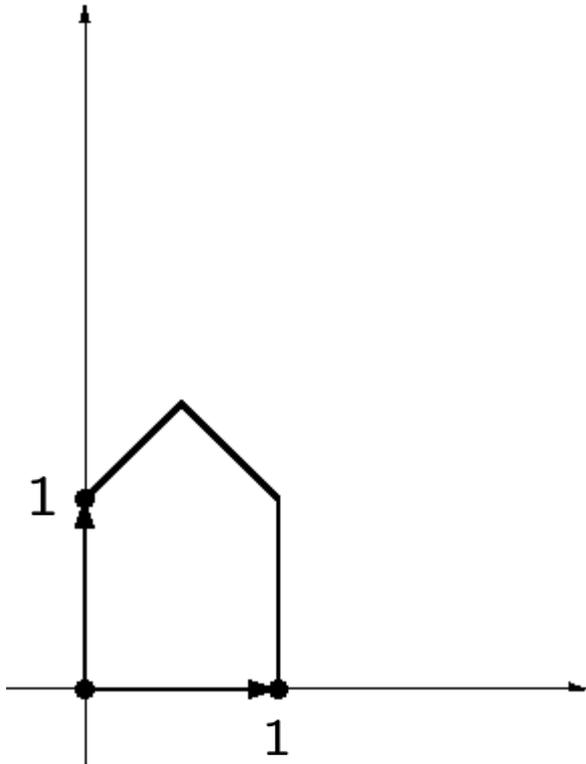


$$R = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

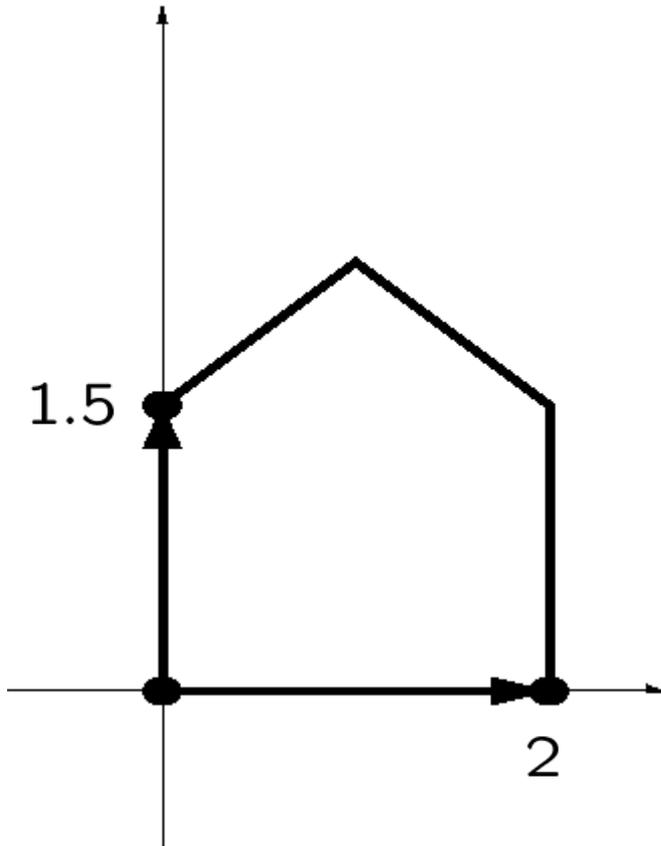
$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -\sin \theta \\ \cos \theta \end{bmatrix}$$

2차원 축소 확대



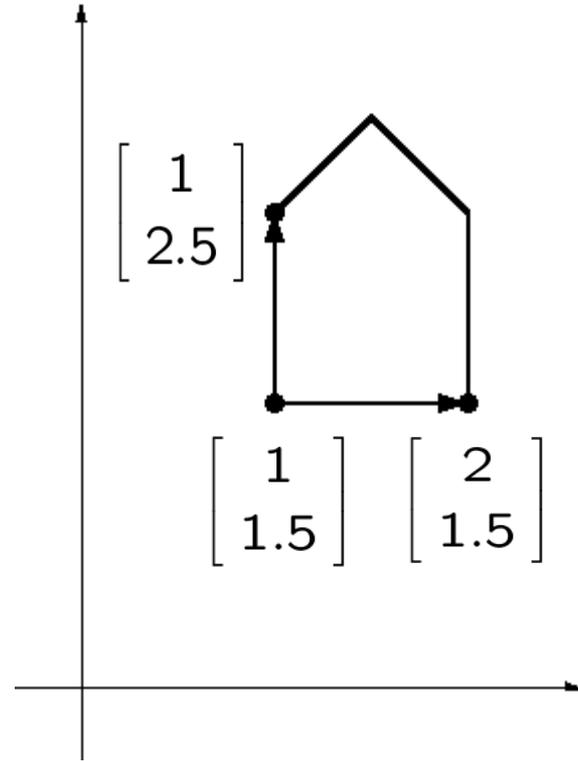
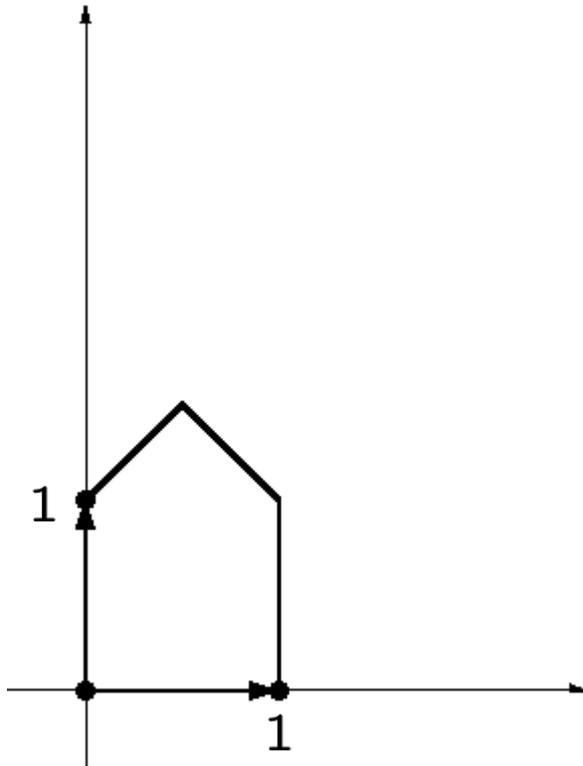
2차원 축소확대



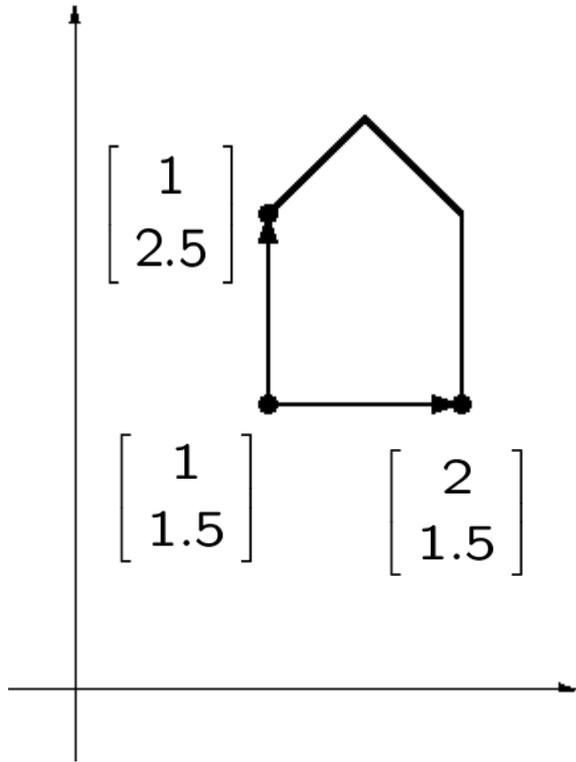
$$S = \begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1.5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 1.5y \end{bmatrix}$$

2차원 평행이동



2차원 평행이동



$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1.0 \\ 1.5 \end{bmatrix}$$
$$= \begin{bmatrix} x + 1.0 \\ y + 1.5 \end{bmatrix}$$

2차원 변환

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

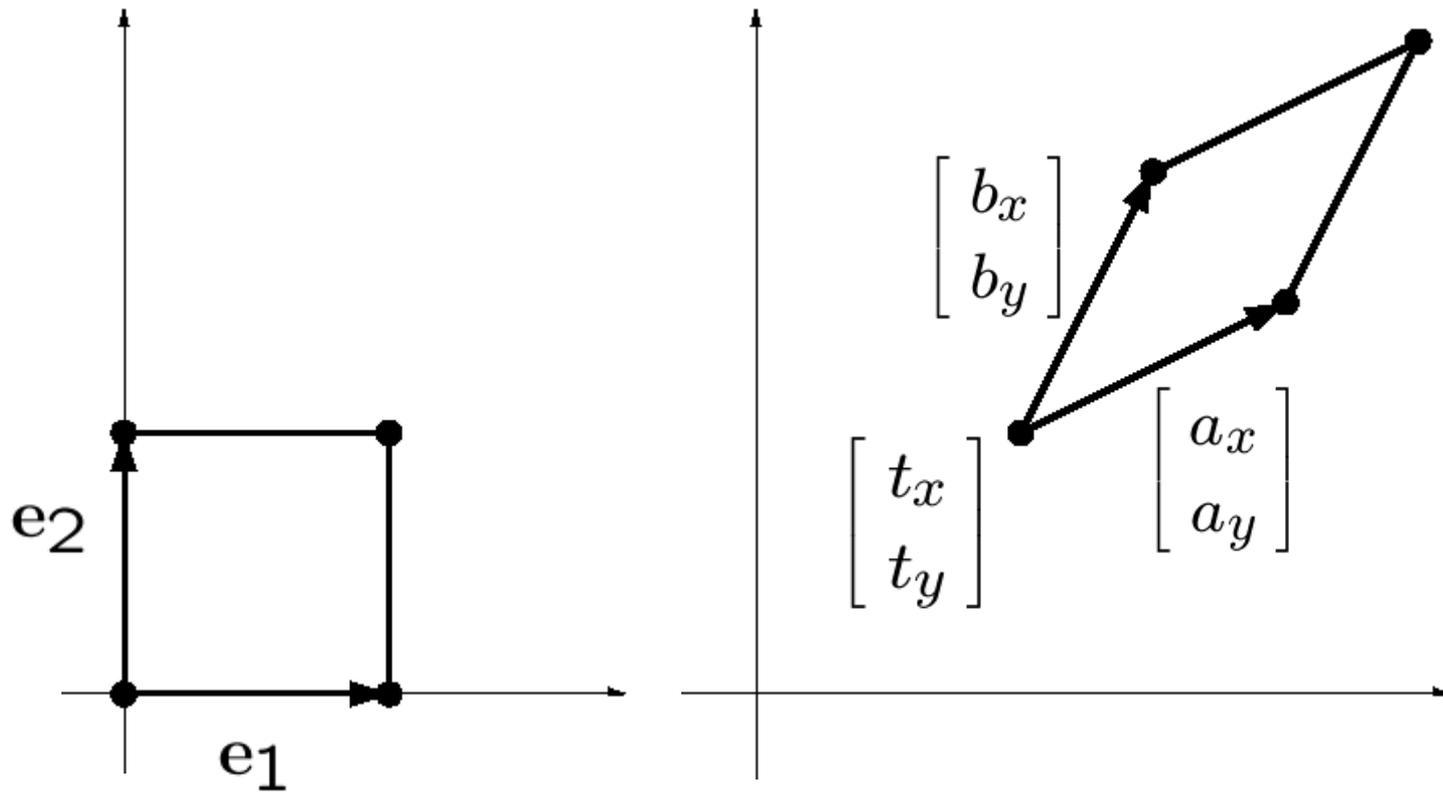
Homogeneous 좌표계

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \mathbf{1} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \mathbf{1} \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ \mathbf{1} \end{bmatrix}$$

2차원 변환



2차원 변환

$$\begin{bmatrix} \hat{x} \\ \hat{y} \\ \mathbf{1} \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} a_x & b_x \\ a_y & b_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

2차원 변환

$$\begin{bmatrix} t_x \\ t_y \\ 1 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

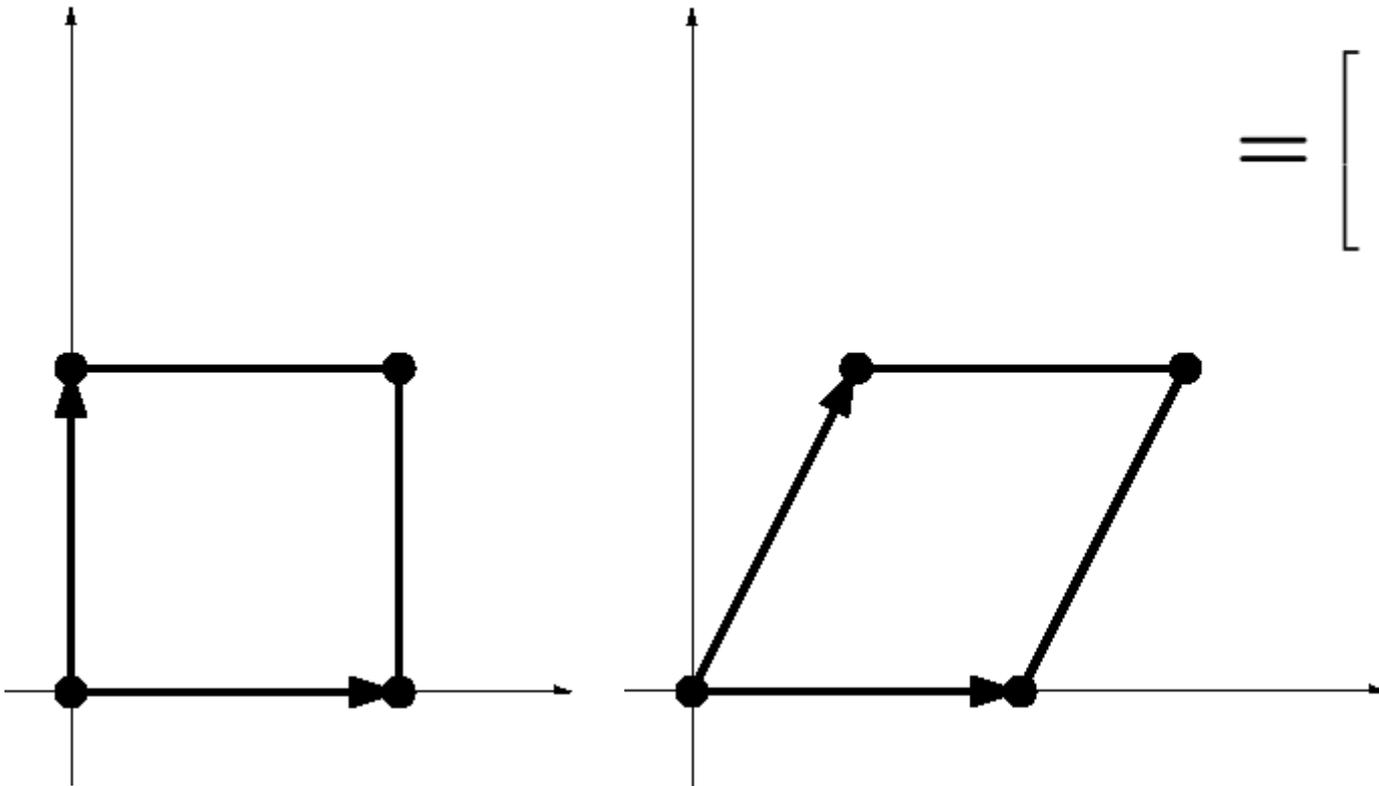
$$\begin{bmatrix} a_x \\ a_y \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} b_x \\ b_y \\ 0 \end{bmatrix} = \begin{bmatrix} a_x & b_x & t_x \\ a_y & b_y & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

2차원 x-shearing

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

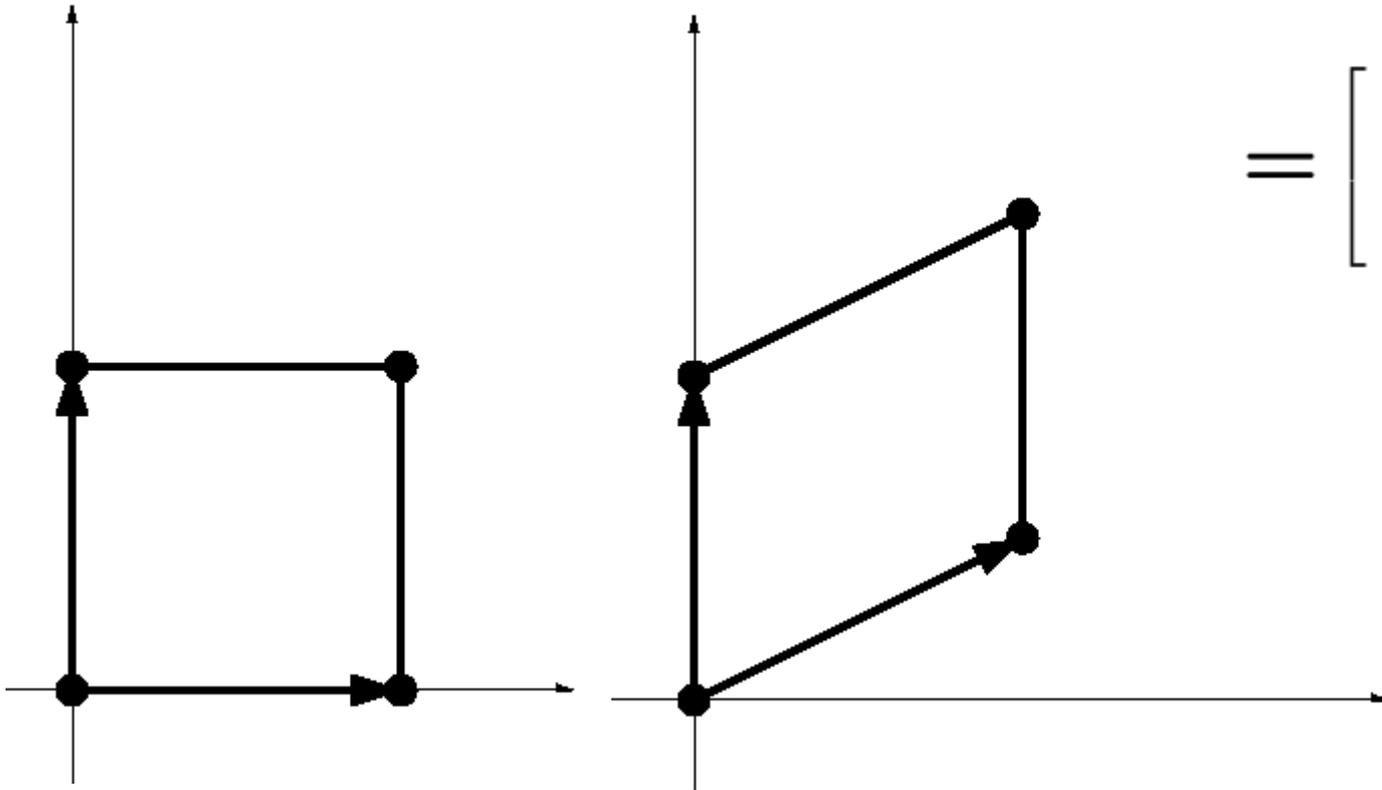
$$= \begin{bmatrix} x + \alpha y \\ y \end{bmatrix}$$



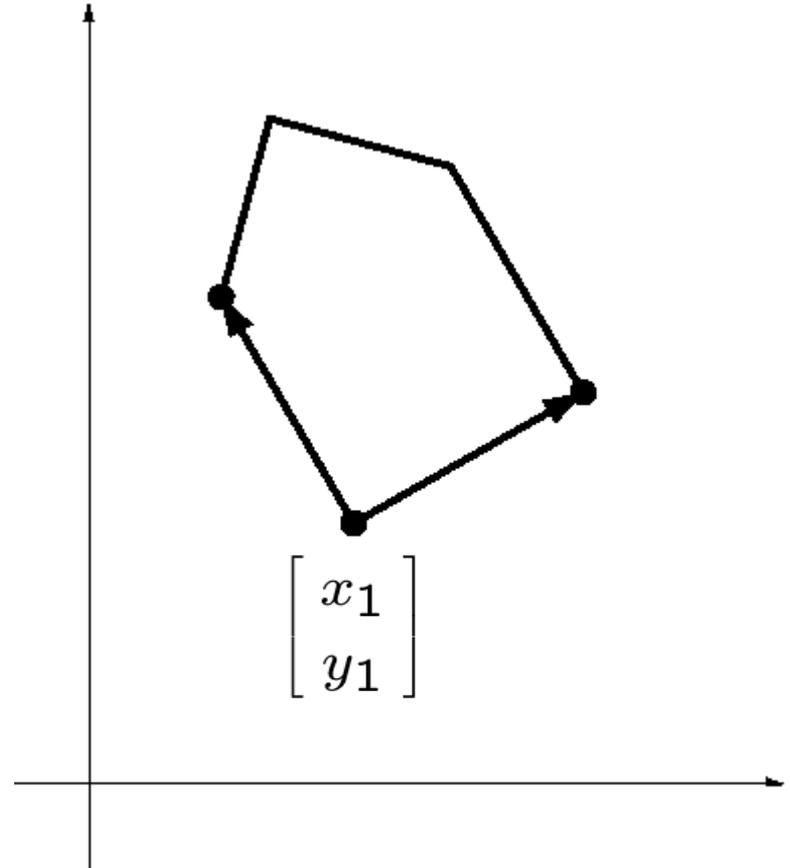
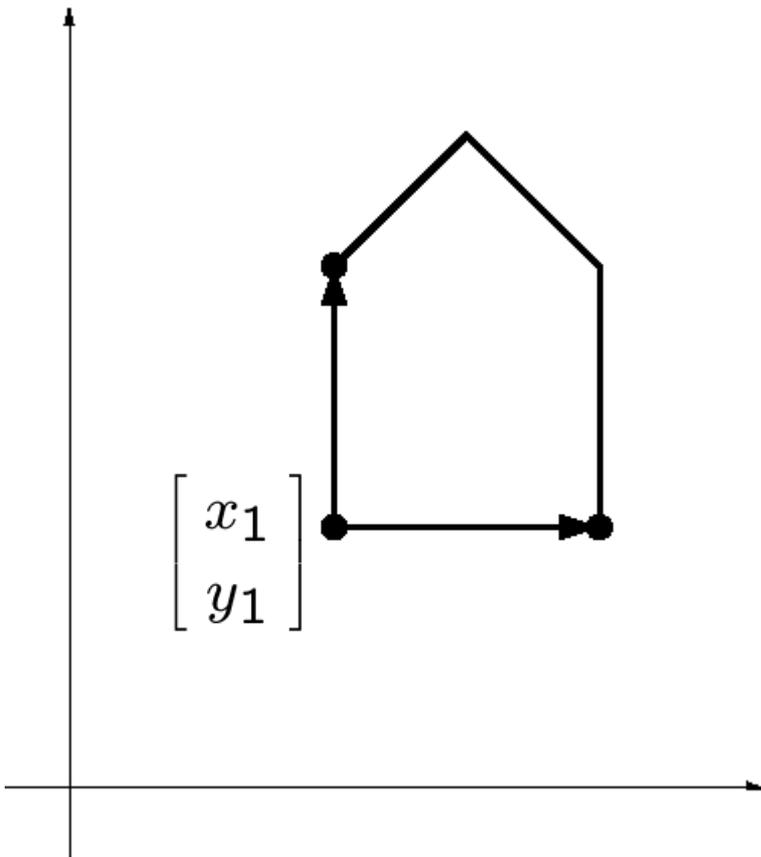
2차원 y -shearing

$$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \beta & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

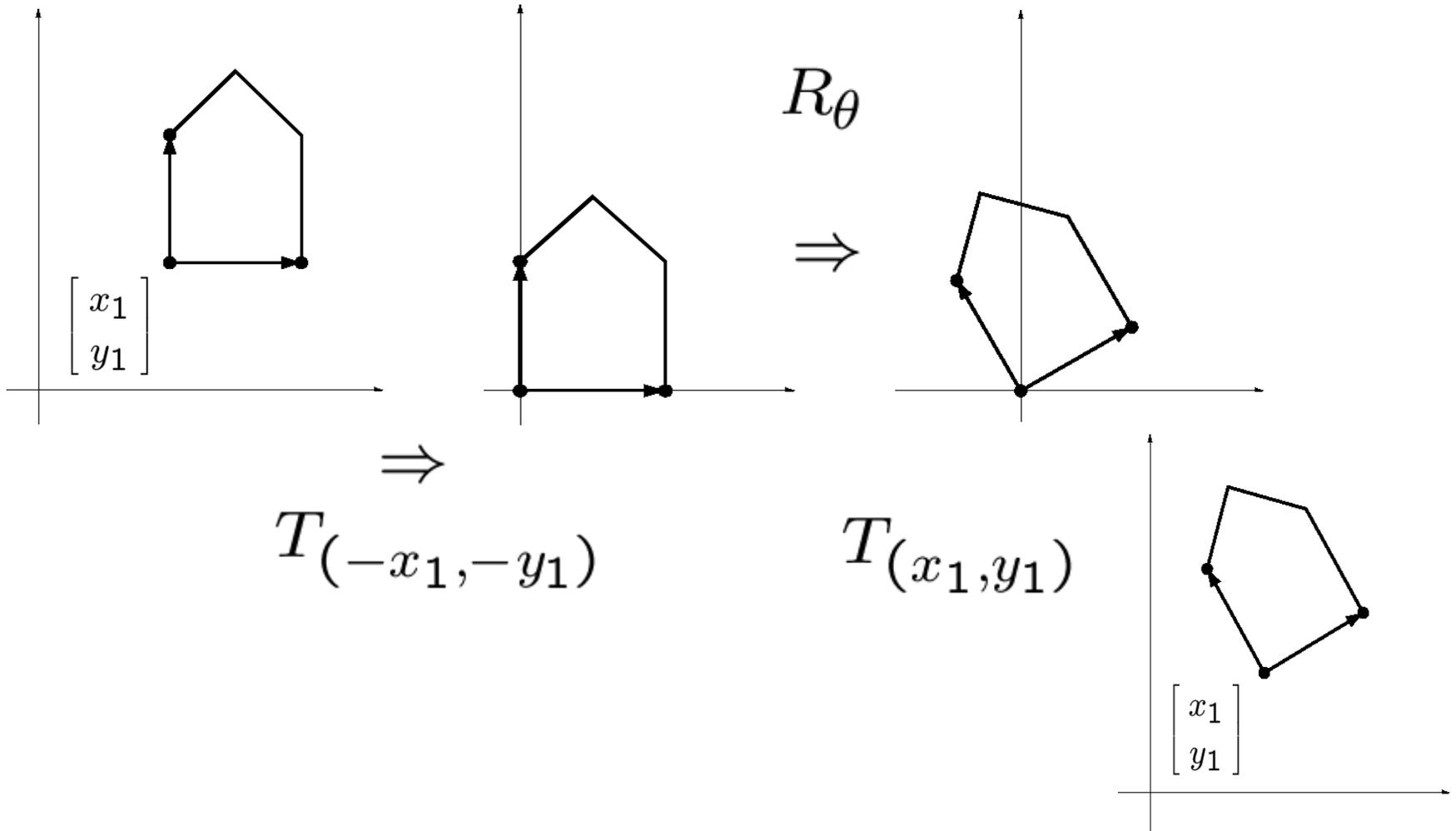
$$= \begin{bmatrix} x \\ \beta x + y \end{bmatrix}$$



2차원 일반회전



2차원 일반회전



2차원 일반회전

$$\begin{aligned} & \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_1 \\ 0 & 1 & -y_1 \\ 0 & 0 & 1 \end{bmatrix} \\ = & \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta & -\sin \theta & -x_1 \cos \theta + y_1 \sin \theta \\ \sin \theta & \cos \theta & -x_1 \sin \theta - y_1 \cos \theta \\ 0 & 0 & 1 \end{bmatrix} \\ = & \begin{bmatrix} \cos \theta & -\sin \theta & x_1(1 - \cos \theta) + y_1 \sin \theta \\ \sin \theta & \cos \theta & y_1(1 - \cos \theta) - x_1 \sin \theta \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$